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ABSTRACT

In September 1979, the Mathematical Association in England held a weekend seminar on the scope of Applied Mathematics at A-level, and a subcommittee was established to consider the topic at more length. This paper is the first product of the subcommittee's deliberations. Sections 1 and 2 describe the background to current A-level courses: (1) who takes A-level Mathematics, and what do they go on to? and (2) a resume of current A-level syllabuses. Section 3 discusses the Applied Mathematics which should form a compulsory component of a single-subject A-level Mathematics exam, and section 4 presents a proposal for a single-subject A-level Mathematics course. An appendix contains data on students who take the A-level course.

(MNS)

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Mathematics

at

A-Level

A discussion paper
on the Applied content

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This booklet was prepared for the Mathematical Association by members of a sub-committee appointed by the Teaching Committee.

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MATHEMATICS AT A-LEVEL : a discussion paper on the Applied content

In September 1979 the Mathematical Association held a weekend seminar on the subject "Applied Mathematics at A-level - what should it be?". As a consequence a sub-committee of the Teaching Committee of the Association was set up to consider the matter at more length. This paper is the first product of the sub-committee's deliberations; we hope it will provoke further constructive discussion and feedback.

Sections 1 and 2 of this report describe the background to the current A-level courses available; sections 3 and 4 contain the sub-committee's outline proposals for the future. It should be noted at the outset that these proposals are limited to the consideration of Mathematics as a single-subject A-level.

1. Who takes A-level Mathematics, and what do they go on to do afterwards?

We have looked at these questions from two viewpoints:

- i) the post A-level destinations of A-level Mathematics candidates;
- ii) the other A-level subjects taken with Mathematics.

1.1 The detailed findings are in Appendix A. The charts in figure 1 summarise the destinations of those school leavers who passed an A-level Mathematics examination as percentages of the national entry from schools in 1976 (it also shows the approximate 1/3 who failed A-level Mathematics). They do not include entries from Further Education Colleges (see Appendix A) because such data are not readily available. This is unfortunate since the F.E. entry is increasing and there are clear implications for the Applied content of A-level, since on the one hand a larger proportion of F.E. students go on to "other employment", and on the other, one suspects that the pattern of other A-levels taken with Mathematics would be more vocational than in schools.

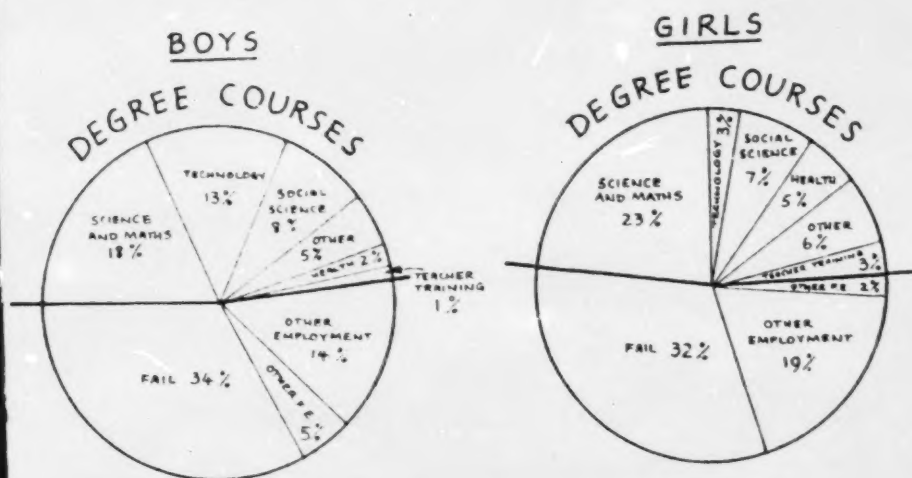


Figure 1 (Source: DES Statistics)

These figures, albeit approximate, speak for themselves: there is a wide variety of destination, with no one career attracting a large proportion and with a large percentage for whom a broad A-level mathematics syllabus would seem most appropriate.

National figures are not available for combinations of A-level subjects including Mathematics. However, the breakdown of entries to the JMB was obtained for 1979. Table 1 shows this breakdown for 11,300 candidates who took single-subject A-level Mathematics (including SMP and MEI inter-board candidates). Table 2 shows the breakdown for a further 3,500 candidates entered for JMB A-level Pure Maths and Statistics.

Other Subject	% of mathematics candidates taking this subject
Physics (including Nuffield Physics)	67.2%
Chemistry (including Nuffield Chemistry)	49.5%
Further Mathematics	14.3%
Biology (including Nuffield Biology)	12.5%
Geography	9.5%
Economics	8.8%
English Literature	3.4%
History	3.0%
French	2.5%
Geometrical and Engineering Drawing	2.0%
Art	1.7%
General Studies	60.2%

Table 1

Other subject	% of PMS candidates taking this subject
Economics	30.5%
Biology (including Nuffield Biology)	27.4%
Chemistry (including Nuffield Chemistry)	25.2%
Geography	21.6%
English Literature	12.1%
History (including Ancient History)	11.2%
French	10.7%
Physics	8.2%
German	3.6%
Art	2.0%
General Studies	59.9%

Table 2 (Source: JMB)

Although Physics (and to a lesser extent Chemistry) dominates the entries with Mathematics in Table 1, there are substantial numbers of candidates taking a wide variety of other subjects - indicating again the need for a broad Mathematics syllabus.

This diversity is even more marked in Table 2.

2. A resumé of current A-level syllabuses

The Present

Taking the current situation as that indicated by the GCE Boards in their 1982 syllabus booklets, it appears that the pattern of A-level Mathematics examinations is different in each Board and considerably different in some cases. For example, Pure Mathematics is not currently offered by all the GCE Boards. All the Boards do offer a subject called Mathematics, though as can be seen from the table below some Boards have alternative syllabuses and some have a range of options embodied within the examination structure.

Single Subject Mathematics

(excluding the subjects Pure Mathematics and Applied Mathematics)

AEB	Mathematics (Pure and Applied), Mathematics (Alternative Syllabus) Also offered Mathematics (Pure with Statistics), Mathematics (Pure with Computations)
JMB	Mathematics Syllabus A, Mathematics Syllabus B Also offered Pure Mathematics and Statistics
London	Mathematics Syllabus B, Mathematics Syllabus C*, Mathematics Syllabus D* Also offered Pure Mathematics and Statistics
Cambridge	Mathematics Syllabus A, Mathematics Syllabus B
Oxford	Mathematics
SUJB	Mathematics
Ox & C	Mathematics
WJEC	Mathematics
SMP and MEI Mathematics are available through every Board	

* To be withdrawn after January 1984

One interesting point, not shown in the table, is the state of Applied Mathematics which was apparently in decline a few years ago with most Boards withdrawing this subject from their list. Now all Boards offer this except the three Oxbridge Boards. Applied Mathematics in this context means Mechanics.

As might be expected the kind of mathematics offered under the title Pure Mathematics does not vary a great deal. If a candidate has taken Pure Mathematics with any of the GCE Boards at A-level, then, according to the examination syllabuses published, the candidate could not be expected to know any mechanics or statistics (unless he is an AEB candidate he could not be expected to know any probability either). Similarly users seeing a certificate bearing the subject title Pure Mathematics with Statistics as

with London and JMB or Mathematics (Pure with Statistics), Mathematics (Pure with Computations) and Mathematics (Pure and Applied), as used to distinguish between the AEB syllabuses, might reasonably be expected to know broadly what kind of Mathematics the candidate has studied, provided in the last case that one understands that 'applied' in this context means 'mechanics'.

The problem then, if there is one, relates to the remaining subjects certificated as Mathematics and referred to as 'Alternative syllabus' or 'Syllabuses A, B, C or D'. Here users who believe that the general title Mathematics means that candidates have studied a fair spread of Pure Mathematics and applications may well be disappointed, for it could mean that candidates have studied in addition to their Pure Mathematics both, one or neither of the areas mechanics or statistics.

In detail the situation is as follows:

- i) If candidates have done SMP, JMB (B), London (C) or SUJB then it is expected that mechanics, probability and statistics will be studied.
- ii) If they have done MEI, Cambridge (A), Oxford, O + C, SUJB or WJEC there is a choice in applied mathematics between answering questions on either or both of the alternatives mechanics and probability-and-statistics, though with MEI all candidates are expected to know some probability.
- iii) If they have done Cambridge (B) or AEB (Alternative Syllabus) the system of options makes it possible to have offered either, both, or neither of the alternatives mechanics and probability-and-statistics though the candidates should know some probability.
- iv) If they have done JMB (A), London (B or D), (or AEB Pure & Applied), they will have studied mechanics but no statistics, though all except the JMB syllabus include some probability.

The future

During the next few years subsequent to 1982, most Boards will probably be revising their syllabuses in order to include the national common core. Following the joint statement by the Standing Conference on University Entrance and the Council for National Academic Awards concerning a minimal core syllabus for A-level Mathematics a few years ago, representatives of the GCE Boards drew up a common core (in fact a list of 27 topics) which they felt should be included in all A-level Mathematics syllabuses. The proposals they made relate to A-level syllabuses which have a substantial element of Pure Mathematics such as *Pure Mathematics*, *Mathematics Pure and Applied*, *Pure Mathematics with Statistics* and *Pure Mathematics with Computations*. They were not however concerned with syllabuses in *Further Mathematics*. Clearly, even if all Boards accept the recommendations, it will not be possible for Boards to incorporate the proposed core material into all their syllabuses immediately. However it is expected that all Boards will include the national core in all their A-level Mathematics syllabuses by 1986.

The suggested core (1) is entirely composed of Pure Mathematics and is approximately 40% of an A-level Mathematics syllabus. The Boards felt that there would be little to be gained from the development of a national core if it were not adequately tested in all Boards' A-level examinations in Mathematics. At present it is envisaged that each Board will examine

the national core in its own way but it is assumed that each will try to ensure that questions set directly on the core will carry not less than 40% of the total marks for any A-level examination in Mathematics.

The content of the core includes a number of algebraic topics, some co-ordinate geometry, trigonometry and work on vectors, functions and calculus. There are a number of other (pure) topics, 11 in all, which it was hoped that all Boards would include in their A-level Mathematics syllabuses in addition to the common core.

Obviously the existence of a national common core will have a direct impact on what can be assessed in the remaining 60% of any Mathematics examination and will certainly have substantial implications for any future developments in Mathematics at A-level.

To date there are detailed schemes only from the London Board and the JMB which incorporate the common core into single subject syllabuses.

The London Board examination consists of two papers on the (pure) core, plus a further paper which may be chosen from three alternatives - (a) mechanics and probability, (b) further pure, or (c) statistics. The titles of the subjects are then respectively Mathematics, Pure Mathematics and Pure Mathematics with Statistics.

The JMB scheme has one paper on the core, followed by a choice of one of (a) further pure, (b) Mechanics, (c) Statistics, or (d) Mechanics and Statistics. Again there are appropriate titles.

3. The Applied Mathematics which should form a compulsory component of a single-subject A-level Mathematics exam.

3.1 The majority of candidates in the future will no doubt follow a single subject A-level containing both pure and applied mathematics; it is with this majority that our proposals are concerned.

It is necessary here to draw a distinction between two quite different ways in which applications may be used in mathematics teaching: the small-scale use, to illustrate or motivate the development of what is essentially a "pure" topic; and the extended development of a whole area of application for its own sake. It will be clear from what follows in section 4 that we should like to see the motivational use of applications employed as often as possible throughout the course, but in this section we shall now concentrate exclusively on those topic areas we feel should be developed in some depth.

3.2 Possible major areas of application for inclusion

Traditionally, as far as English mathematical education at school level goes, 'applied maths' has meant mechanics. More recently, there has been pressure for the teaching of more statistics and probability in maths courses. Another area of application which has been introduced at A-level has been electricity. No serious attempt has been made to develop any extended work within a mathematics course on the application of mathematics to economics, geography etc., nor does there seem to be any great demand for such a development.

We feel that teaching one or more applications in an extended way fulfils two major functions: students see the power of mathematics as it is used in other subjects; they learn the skills of identifying the important variables, postulating relationships between them and assessing the implications for the real world of results from mathematical models. At the same time the applications can provide an ideal vehicle within which to develop other, 'pure' topics in the course. In choosing which applications to include, we paid particular attention to these functions, as well as considering the possible relevance to students' career or higher education intentions.

It has been suggested that considerably more attention should be paid to teaching students the modelling process. We feel that at this level, with the inevitable constraint of time, our first priority should be to build on the experiences that they have already, using established models in the most positive way we can - underlining the assumptions and limits of validity.

There is little doubt that mechanics and statistics are the two most well-established application topics. Mechanics gives scope for the development of vectors, which can be linked to linear algebra and geometry as well as motivation and examples in the development of calculus. Furthermore, we all have so much experience of moving bodies in the world about us. Equally, statistics and probability provide opportunities for development of algebra and calculus, as well as numerous links to other subjects in the curriculum and everyday life.

The basic question is, then, should we teach mechanics, statistics, or both to each student?

We feel that all those who complete an A-level Mathematics course should have studied something from both areas - that they should experience both deterministic modelling (in mechanics) and non-deterministic (in statistics).

3.3 Option schemes

Several of the most recent syllabuses (eg London and JMB referred to earlier) have sought to accommodate the demand for the teaching of several major areas of application (principally mechanics and statistics) by introducing option schemes. The major attractions of options include:

- i) one application can be studied in some depth rather than several more shallowly. If the "core" is to be 40% of the course, two applications would only have 30% each - not enough, runs the argument, to develop either far enough to be worthwhile;
- ii) students with career intentions which require a study of a particular option may choose it and not 'waste' time on any others;
- iii) there are not always teachers in a school competent to teach both, and many teachers prefer to teach only the topics they favour. A syllabus with options can be used to overcome these limitations of staffing.

Arguments against options include:

- iv) there are powerful educational reasons for keeping the curriculum as wide as possible at this level: we would argue that a study of motion and its causes, and an introduction to statistical methods which are being used in so many different disciplines these days, should form compulsory parts of A-level maths for everyone, even if their career choice is already made;
- v) our analysis in section 1 shows clearly that the largest group of A-level mathematics students have career intentions (or a lack of them) which do not commit them solely to either mechanics or statistics. We feel therefore that these students should meet both these topics at A-level;
- vi) in practice, few schools can afford the price of offering options effectively at A-level, in terms of the shortage of maths teachers and the small size of maths classes. Allowing schools to limit the applied topics they offer is likely to reinforce in the present, and carry forward to the future, the unfortunate situation in which teachers' conservatism limits students' choice - a situation all too prevalent;
- vii) in operation, an option scheme would almost certainly mean that 'applied' maths was taught separately from 'pure'. We should prefer the two to be as firmly intertwined as possible, as we shall show in section 4. We should like to see as much of the mathematics of the course as possible grow out of interesting contexts, experiments and situations.

3.4 Conclusions

Naturally, our discussions have been coloured by the current debate about a 'common core' of pure topics for all A-level maths syllabuses. In the outline for an A-level course which follows in section 4, we have included all the topics listed in the latest 'core' proposals, though they are not grouped together, nor would we intend them to be examined separately. The distinctive features of the course are:

- i) both statistics and mechanics are integral components;
- ii) 'pure' topics are motivated by, or grow from, applications wherever possible;
- iii) no topic is optional, and the exam would be so designed as to deter teachers and students from not covering the entire syllabus;
- iv) the assumptions and limitations of the models developed are to be made as explicit as possible.

4. A proposal for a single-subject maths A-level course

In what follows, the degree of detail varies with our need to make clear how individual topics link and develop. Although set out in conventional "syllabus" form, we allowed ourselves a degree of informality to help emphasise the continuity and coherence between one topic and another and between "pure" and "applied", that we are trying to foster.

Our brief was to examine the Applied content of A-level, but the following proposal is a complete course because we feel that applied and pure cannot be separated: one motivates the other, the latter provides the tools for the former.

Those parts of the course which are our extended developments of mechanics and statistics are in sections 7 and 8.

Although we have not mentioned computers, nor the use of calculators explicitly, we intend the flavour of the whole course to be affected by their presence: the availability of quick generation of numerical results both directly and using iterative methods, and the use of graph-plotters and graphical displays can add to understanding and appreciation. Equally, the process of writing programs can in many topics give new insight into the mathematics involved.

Topic

Notes

1.1 Modelling and functions

A limited survey, confined to looking at data from a variety of contexts, and including the use of tables, graphs and algebraic formulae.

.. developing from this

1.2 Function: domain and range; illustrations by mapping diagram and graph. Particular functions and their graphs: algebraic, circular, log and exponential. Graph sketching; simple transformations of graphs; symmetry; odd, even and periodic; composite and inverse.

Understanding of the relationship between a graph and the assoc. algebraic relation. In particular, ability to sketch $y = x^n$ for integral and simple rational n , $ax + by = c$, $x^2/a^2 + y^2/b^2 = 1$. Geometrical properties of the ellipse, parabola and hyperbola excluded. Transformations $y = af(x)$, $y = f(x) + a$, $y = f(x - a)$, $y = f(ax)$.

.. also

1.3 Trigonometric functions of angles of any magnitude. Circular measure, $s = r\theta$, $A = \frac{1}{2}r^2\theta$. The addition formulae, double angle, factor formulae. Expression of $a\cos\theta + b\sin\theta$ in form $r\sin(\theta + e)$. Period, amplitude, phase. Solution of simple equations; inverse trig. functions.

.. and still related as far as possible to useful models

2. Addition, subtraction, multiplication and division of polynomials, including addition of rational forms; factor theorem for polynomials. The quadratic function, quadratic equations and inequalities; solution of other simple polynomials; partial fractions.

Mention of complex numbers but little development (see Common Core proposals).

Situations where limits,
sequences and series crop up,
leading to

3. Inductive definition of
sequence. Sigma notation;
summation of simple series (AP
and GP; use of formulae for

$\sum_{k=1}^n k$
 $\sum_{k=1}^n k^2$ for $k=1,2,3$). Sum to
infinity of GP.

Rates of change in models from 1,
leading to

4.1 Derivatives of functions of
one variable; differentiation of
products, quotients, composite
functions and functions defined
implicitly. Application of dif-
ferentiation to gradients of
graphs, maxima and minima. Second
and higher derivatives

.. and

4.2 Approximation: tangents to
a curve and their equation, Taylor
approximation to simple functions;
Newton-Raphson method of solution
of equations.

Situations where area under a
graph is of interest, leading
to

5.1 Integration: as anti-differ-
entiation- standard integrals,
integrations by parts, simple sub-
stitution. Integration as
summation; application to area,
volume, centre of mass. Numerical
methods: trapezium and Simpson's
rules

.. or where, given the rate
function $f'(x)$, we wish to find
 $f(x)$

5.2 Differential equations:
simple step-by-step solution of
 $dy/dx=f(x,y)$; families of solution
curves. Explicit solution of
first-order equations with
separable variables.

Derivatives of x^n , $\sin x$, $\cos x$, $\tan x$,
 $\sin^{-1}x$, $\tan^{-1}x$, e^x , a^x , $\ln x$.

Integrals of x^n , $\sin x$, $\cos x$, e^x , $1/x$,
 $1/(1+x^2)$, $1/\sqrt{1-x^2}$.

The relation with corresponding tech-
niques of differentiation should be
explicit.

Simple experiments involving moving bodies

6.1 The idea of a position vector; position vectors as functions of time:

$$\underline{r} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \text{ in 2D}$$

eg. One ball bearing dropped vertically as another is projected horizontally from the same height (listen as they hit the floor together; take multiflash photo to analyse).

Numerical investigation of average velocities, leading to

6.2

$$\underline{v} = \frac{d\underline{r}}{dt} = \begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix}, \quad \underline{a} = \frac{d\underline{v}}{dt} \text{ for}$$

Where $\underline{r}$, $\underline{v}$, $\underline{a}$ are expressed as functions of time in component form.

the motion of particles in the plane (among other paths, motion in a circle and under constant acceleration are required).

6.3 Position vectors, the basic triangle $\underline{AB} = \underline{b} - \underline{a}$; the ratio theorem; vector equation of the line $\underline{r} = \underline{a} + k(\underline{b} - \underline{a})$.

All this to be done on plain paper with arbitrary vectors, and on squared paper with unit vectors $\underline{i}$ and $\underline{j}$ and columns $\underline{r} = \begin{pmatrix} x \\ y \end{pmatrix}$, to get

the student used to the ideas by encouraging him to move from point to point on the paper.

6.4 Scalar product; angles between lines.

$\begin{pmatrix} a \\ b \end{pmatrix}$ perpendicular to $ax + by = c$

7. Mechanics

The mathematical work on mechanics is restricted to those topics for which pure mathematical tools are available (or will be developed) for their discussion. This means that the models are very simple. Nevertheless, using only particles, and restricting motion to two dimensions, there is a rich supply of mechanical phenomena which can be developed and illustrated in the classroom. The syllabus introduces concepts which are demonstrably useful when simple experiments and problems are discussed and have obvious importance outside the classroom.

Discussion of the conditions and limitations of validity is considered very important.

Topic	Notes	Links
7.1 Newton's laws of motion for a particle.	One or 2D only; the second law as $\frac{d}{dt}(mv) = F$ $F=ma$ problems solved by drawing vector polygons and in components; simple experiments.	Vectors and their derivatives.
Statics of particles.	As a particular case of motion.	
Laws of friction; Hooke's law; particle attached to an elastic string (in 1D); small oscillations.	Experimental evidence; discussion of validity. Use of energy. S.H.M.; the equation $\ddot{x} + \omega^2 x = 0$	Integration of $\ddot{x} + \omega^2 x = 0$ as solution of two first order differential equations.
Motion of a particle under gravity.	2D motion may be assumed.	Integration; energy.
Motion of a particle in a vertical circle.	Use of Conservation of energy.	
7.2 Conservation of mechanical energy.	Restricted to 2D conservative forces and derived by integration of Newton's 2nd law; discussion of situations where the principle does not apply.	Scalar product; integration.
7.3 Conservation of linear momentum for two particles in direct impact; concept of impulse; restitution.	Derivation by integration of Newton's 2nd law. Use of simple experiments.	Integration.
8. Probability and Statistics		

In this section of the syllabus we are attempting to convey the notions of modelling non-deterministic situations and of using data to reflect on the model (early stages of inference). We aim to expose pupils to practical problems and data involving random variation and uncertainty, to introduce probability as a measure of uncertainty and probability theory as the mathematical model for combining uncertainties. Sampling comes in as a means of both obtaining information about a practical problem and, by use of probability theory, as information from which one can draw inferences.

Since the syllabus is part of a mathematics A-level it concentrates on the mathematical aspects of elementary probability and statistics. This does not lessen the importance of doing practical and project work, but it does explain the omission of such topics as practical problems of data collection, carrying out surveys, reading tables, simple aspects of experimental design,

elementary time series, moving averages, index numbers, birth and death rates, data sources and many other items that might be included if the syllabus were to be seen largely as a service to other subjects such as economics, geography, biology and chemistry.

Candidates should appreciate the aims, scope and limitations of statistical procedures. Questions may involve a discussion of the practical implications of statistical conclusions.

Topic	Notes	Links
8.1 Descriptive Statistics		
Frequency and relative frequency distributions. Histograms. Sample mean, median, percentiles, variance and standard deviation.	To illustrate variation in practical problems. Calculated for grouped and un-grouped data.	Use of the Σ notation, algebraic manipulation, linear interpolation.
8.2 Probability		
Relative frequency of an event and its relationship to probability.	Probability as a measure of uncertainty.	The idea of a limit.
Sample space. An event as a subset of the sample space.		
Equally likely events.	Including simple combinatorial methods.	The notation $\binom{n}{r}$. Permutations and combinations.
The addition law for probabilities.		
Conditional probability and the multiplication law; independent events.		Sets and set notation. Union and intersection of Sets. Simple Boolean algebra.
Applications of these laws to two or more events.	Use of appropriate diagrams such as Venn diagrams, tree diagrams and Karnaugh diagrams.	
8.3 Probability distributions		
Discrete and continuous random variables.		A random variable as a function.
Probability functions, probability density functions and distribution functions.	As models to describe patterns of variability.	Standard mathematical functions. Sums of series, including sums to infinity.
Distribution parameters such as the mean, median and variance.		Sums of series and integration.

Discrete uniform distributions.

To model random choice and link with random number tables.

Binomial distribution. The distribution of a proportion.

With emphasis on the assumptions necessary to make these useful models. Pascal's triangle; development of binomial theorem for positive integral index.

Link with Taylor series for $(1+x)^n$ for n rational or negative; general binomial theorem.

Continuous uniform distribution.

Other simple continuous models.

Use of integration to evaluate probabilities and find values of distribution parameters.

Standard functions such as $\exp x$, $\sin x$, polynomials.

The normal distribution as an approximation to the binomial. Use of normal tables.

Including the use of the continuity correction.

Areas under curves.

Expectation. Expectation of a function of a random variable.

The Σ notation and integrals of standard functions used above, including integration by parts.

Expectation of the sum of independent random variables.

Intuitive treatment only.

8.4 Sampling, estimation and hypothesis testing

Population and sample. Practical sampling. Random samples.

Use and implications of the terms 'with replacement' and 'without replacement'.

Modelling a population by a probability distribution.

Using sample statistics to estimate distribution parameters. Unbiased estimators of the mean and proportion.

See 'Expectations' in §8.3.

Hypothesis testing using the binomial distribution as a model (or the normal approximation where appropriate).

Includes the use of the sign test.

Binomial theorem.

Null and alternative hypotheses, critical region, the two types of error and significance level.

Areas under curves. Summation and integration.

Reference

- (1) A minimal core syllabus for A-level mathematics - a joint statement by the Standing Conference on University Entrance (SCUE) and the Council for National Academic Awards (CNAA).

Appendix A

What sort of person takes an A-level Mathematics Course? There are two ways of looking at this from a national point of view:

- (i) by seeing where those who pass go, on leaving this stage of their education.
- (ii) by considering the other A-level subjects the students are taking along with their mathematics.

The source of statistics for first approach is DES Statistics of Education, 1976 as being most recently available at time of writing. In spite of extra detail made available especially by the DES for this purpose it is still not completely possible to get a sufficiently detailed disaggregation in some respects of the data. All the figures quoted should be taken as representing the correct order of magnitude of the results rather than being exact, since a number of assumptions have had to be made.

One thing that must not be forgotten is that, of the students studying A-level mathematics, a large proportion fail. In 1976 of the 58450 entries by boys 19849 (34%) failed and of the 15224 entries by girls 4885 (32%) failed. These data are illustrated in figure 1. They include 14300 boys (5460 failures) and 2770 girls (1020 failures) from Further Education. The figures are for entries rather than students since there are many students who take 'double mathematics' in some form and it is not possible to separate their results out from the nationally published figures. Nevertheless if we take it that roughly one-third of the students fail, and presumably they did not set out on their A-level course with the intention of failing, we have to consider the implication of this for the content of the A-level mathematics syllabuses and, in particular, the applied part of such a syllabus.

Turning our attention to those school leavers who do pass in A-level mathematics (including the double subject) we find that most of them proceed to higher education to study for a degree of one sort or another. There are, however, a large number who leave school at this stage to enter employment, some go into other further education and a small number (proportionately more girls than boys) enter teacher training courses. The data are given in table 1 and illustrated on figure 2. The table gives the percentages of those who passed A-level mathematics; the figure uses these to estimate the proportion of those taking an A-level mathematics course (including failures) proceeding in these various directions. So, for example, figure 2 indicates that approximately 35% of boys taking A-level mathematics courses and 36% of girls taking A-level mathematics courses proceed to study for a degree. This assumes that none of those passing A-level mathematics from an institute of further education proceeds to study for a degree. This is not the case but the numbers so doing are probably relatively small.

Table 1. Future destination of school leavers with at least one A-level pass in Mathematics (1976).

Destination	Boys		Girls	
	Number (1,000's)	Percentage	Number (1,000's)	Percentage
Degree Courses	15.63	70%	4.23	64%
Teacher Training	0.16	1%	0.33	5%
Other further education courses	1.78	8%	0.23	3½%
Employment or not known	4.60	21%	1.82	27½%
		100%		100%

Source: Statistics of Education 1976 DES.

Using table 14 from Volume 2 of Statistics of Education 1976 it is possible to get some indication of the breakdown of types of degree studied by those proceeding in this direction. The DES uses the categories Health, Technology, Agriculture, Science, Social Studies, Language Studies, Arts other than languages, Music Drama and Visual Arts, and other subjects (including not known). The category Science includes the life sciences as well as mathematics and the physical sciences. Unfortunately no breakdown of the figures into these categories was available. Table 2 shows the breakdown of degrees studied by school leavers with at least one A-level in mathematics who proceeded to study a degree.

Table 2. School leavers with at least one A-level in Mathematics entering degree courses: breakdown by subject of degree.

Subject	Boys		Girls	
	Number (1,000's)	Percentage	Number (1,000's)	Percentage
Science	6.16	39.5%	2.25	53%
Technology	4.56	29%	0.26	6%
Social Science	2.66	17%	0.63	15%
Health	0.62	4%	0.47	11%
Other or not known	0.92	10.5%	0.61	14.5%
	15.62	100	4.22	100

Figure 3 shows the implications of these figures when estimated as proportions of those taking an A-level mathematics course. It indicates, for example, that approximately 10% of the boys and 2% of the girls will proceed to study for a degree in technology.

Figure 1. Passes and failures at A-level mathematics: School and Further Education entries (1976).

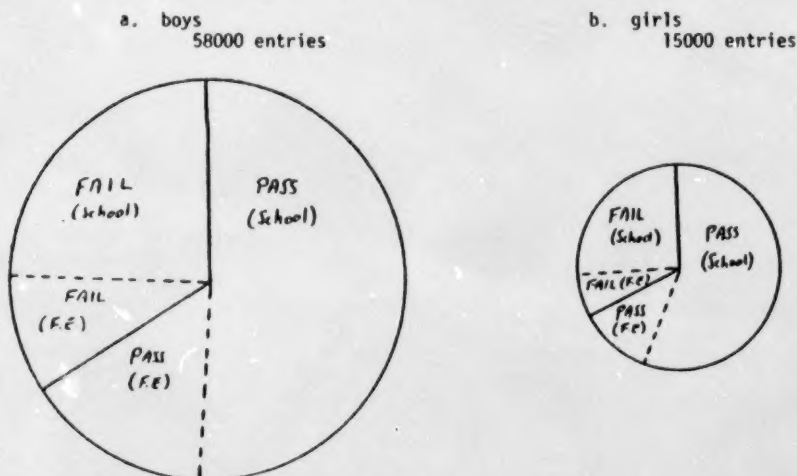


Figure 2. Future destination of school leavers with at least one A-level in mathematics (1976).

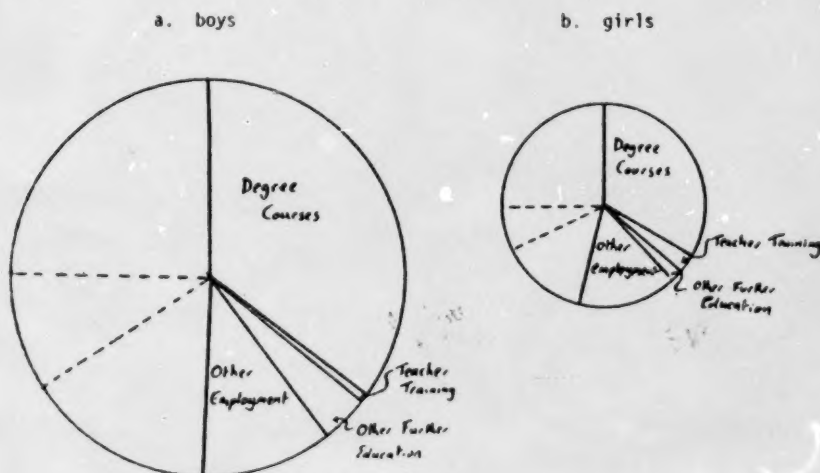
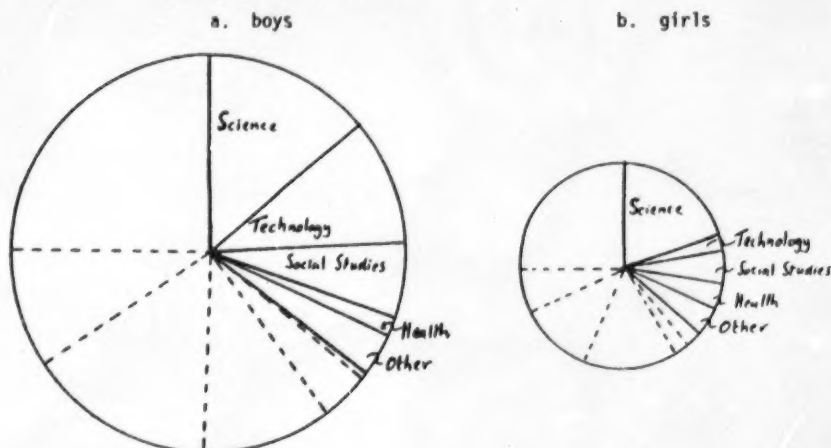


Figure 3. Types of degree studied by school leavers with at least one A-level in mathematics



The second way of analysing the sort of student who takes an A-level mathematics course is to consider the other A-levels being taken at the same time. This will give some measure of the range of interest of such mathematical students. National data are not available, the following come from the 1979 entry for mathematics syllabuses of the Joint Matriculation Board. The JMB has four syllabuses for a single A-level which include some work in applications of mathematics. These are A-level Mathematics Syllabus A, A-level Mathematics Syllabus B, A-level Pure Mathematics with Statistics and A-level Statistics. Syllabus A consists of work in pure mathematics and mechanics. Syllabus B has work in pure mathematics, a smaller amount of mechanics and some statistics. Pure mathematics with statistics is self explanatory, it includes no mechanics. A-level statistics is a relatively new syllabus and has far fewer entries than other syllabuses. It does not contain any specific pure mathematics which can be separately examined. It would not usually be considered as a mathematical qualification. Table 3 shows the number of candidates entering for these examinations in 1979.

Table 3. 1979 entries for selected JMB A-level examinations.

Subject	No. of Entries
Mathematics Syllabus A	8122
Mathematics Syllabus B	1331
Pure Mathematics with Statistics	3544
Statistics	197

The analysis of candidates' other subjects, which is provided by the JMB does not distinguish between syllabus A and B and to this single 'Mathematics' category is also added 1773 candidates who took SMP A-level mathematics through the JMB and 85 candidates who took various MEI A-levels in mathematics through the JMB, giving 11311 entries in all. Table 4 shows the number of these candidates that also took selected other A-level subjects with these figures expressed also as a percentage of these mathematics candidates. These data are also illustrated in Figure 4.

The obvious feature of these data is the large number of candidates taking physics or chemistry along with their mathematics. A more detailed analysis shows that 4417 candidates (39% of those taking mathematics) are taking at least the three A-levels - Mathematics, Physics and Chemistry. It also shows that 1297 candidates (11.5% of those taking Mathematics) are taking Mathematics, Further Mathematics and Physics. Even so there are still large numbers of candidates taking a wide range of other subjects as is also shown in the table.

Table 4. Candidates taking A-level mathematics* through JMB (1979). Breakdown of other subjects taken.

Other Subject	Number of mathematics candidates taking subject	% of mathematics candidates taking this subject
Physics (including Nuffield Physics)	7598	67.2%
General Studies	6805	60.2%
Chemistry (including Nuffield Chemistry)	5601	49.5%
Further Mathematics	1620	14.3%
Biology (including Nuffield Biology)	1411	12.5%
Geography	1075	9.5%
Economics	992	8.8%
English Literature	389	3.4%
History	336	3.0%
French	288	2.5%
Geometrical and Engineering Drawing	231	2.0%
Art	197	1.7%

*A total of 11,311 candidates entering for JMB Syllabus A, JMB Syllabus B A-level mathematics, and the JMB entries for the interboard examinations SMP A-level mathematics and MEI Mathematics, Pure Mathematics and Applied Mathematics at A-level.

Source: JMB.

A very different pattern emerges when we consider the other subjects taken by those candidates entered for the A-level Pure Mathematics with Statistics examination. Details of these 3544 candidates are given in Table 5 and illustrated in figure 5.

Table 5. Candidates taking the JMB A-level Pure Mathematics with Statistics in 1979: Breakdown of other subjects taken.

Other Subject	Number of PMS candidates taking this subject	% of PMS candidates taking this subject
General Studies	2124	59.9%
Economics	1081	30.5%
Biology (including Nuffield Biology)	971	27.4%
Chemistry (including Nuffield Chemistry)	893	25.2%
Geography	766	21.6%
English Literature	428	12.1%
History (including Ancient History)	397	11.2%
French	379	10.7%
Physics	292	8.2%
German	128	3.6%
Art	70	2.0%

Source: JMB

Here no subject dominates in the same manner as physics dominated the previous table. There is a clear difference in that the emphasis has moved to the biological sciences from the physical sciences, and proportionately far more candidates who also take economics. There is no subject combination like Mathematics, Physics and Chemistry of the previous table. A closer analysis shows that of these candidates taking Pure Mathematics with Statistics 622 (17.5%) are also taking Biology and Chemistry and 326 (9.2%) are also taking Economics and Geography.

In taking a global view of the background and the future destinations of students of A-level mathematics courses inevitably some detail is lost. A more detailed survey of the background and intention of pupils in one fairly large sixth form college can be found in Knowles (1980). Nevertheless the general picture is clear in that there is no such thing as a typical mathematics student. There is a wide diversity in their other interests and in their future destinations. This diversity needs to be borne in mind when planning appropriate mathematics courses, particularly courses in applications of mathematics, for these students.

Figure 4. Candidates taking A-level Mathematics* through JMB (1979).
Breakdown of other subjects taken * (See p. 19)

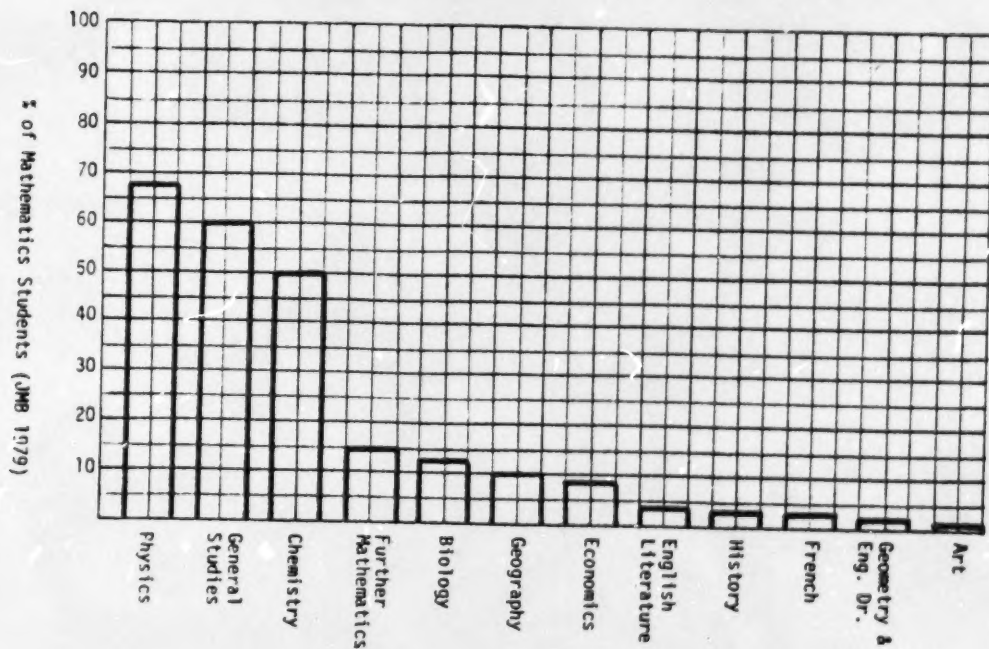
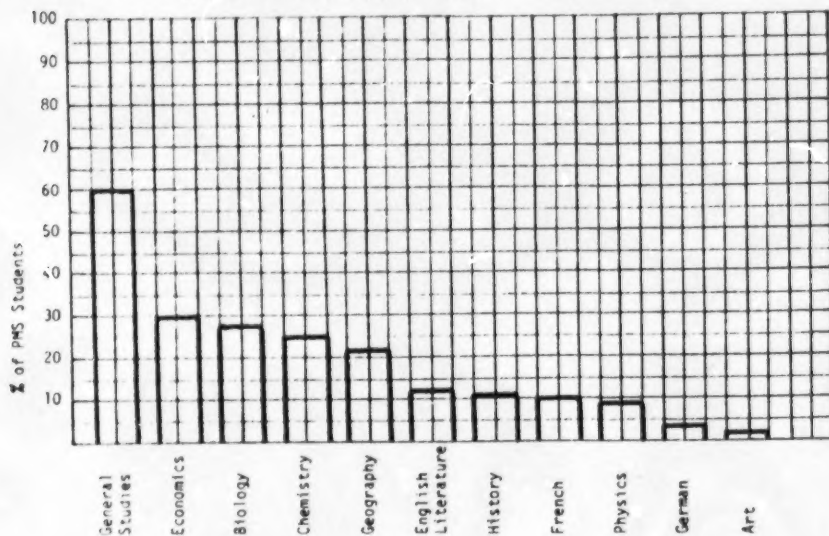


Figure 5. Candidates taking the JMB A-level Pure Mathematics with Statistics 1979:
Breakdown of other subjects taken.



References:

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Volume 2 School Leavers CSE and GCE

Table 12 Subject specialisation and destination of School Leavers with GCE Advanced Level Passes.

Table 14 School Leavers intending to follow full-time degree courses.

Table 16 GCE Advance Level results of students in Further Education.

Table 27 GCE Advanced level results - summer examinations 1976.

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in "A-level Mathematics: The report of the Stoke Rochford Conference, March 1980".

SMP.

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